Chapter 7

Electromagnetic Waves

Overview

- This chapter shows that most of the physics principles related to electric and magnetic fields that we have studied so far can be summarized in only four equations, known as Maxwell's equations.

- We examine the science and engineering of magnetic materials.

Chapter 7:

Electromagnetic Waves
(Chapter 33 in textbook)

7.1. Maxwell's Equations

7.2. Magnetism of Matter

7.3. Electromagnetic waves

7.4. Energy Transport and the Poynting Vector

7.5. Radiation Pressure

7.6. Polarization, Reflection and Refraction

7.1. Maxwell's Equations:

7.1.1. Gauss's Law for Magnetic Particles:

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

$$\Phi_B = \oint \vec{B} d\vec{A} = 0 \text{ (Gauss' law for magnetic fields)}$$

The law asserts that the net magnetic flux F_B through any closed Gaussian surface is zero. Here **B** is the magnetic field. Recall Gauss' law for electric fields:

$$\Phi_E = \oint \vec{E} d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist

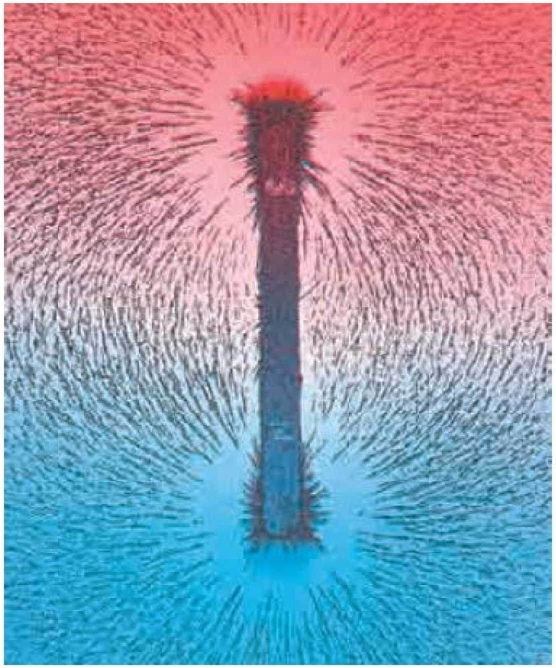
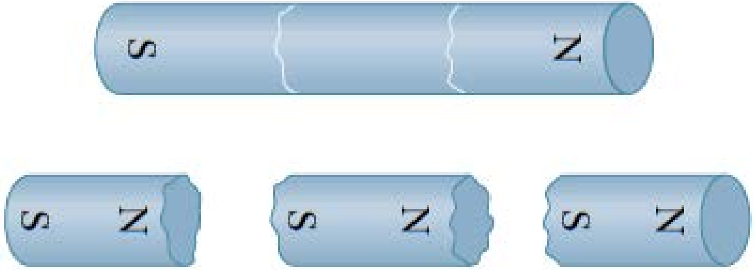


Fig. 32-2 A bar magnet is a magnetic dipole. The iron filings suggest the magnetic field lines. (Colored light fills the background.) *(Runk/Schoenberger/Grant Heilman Photography)*

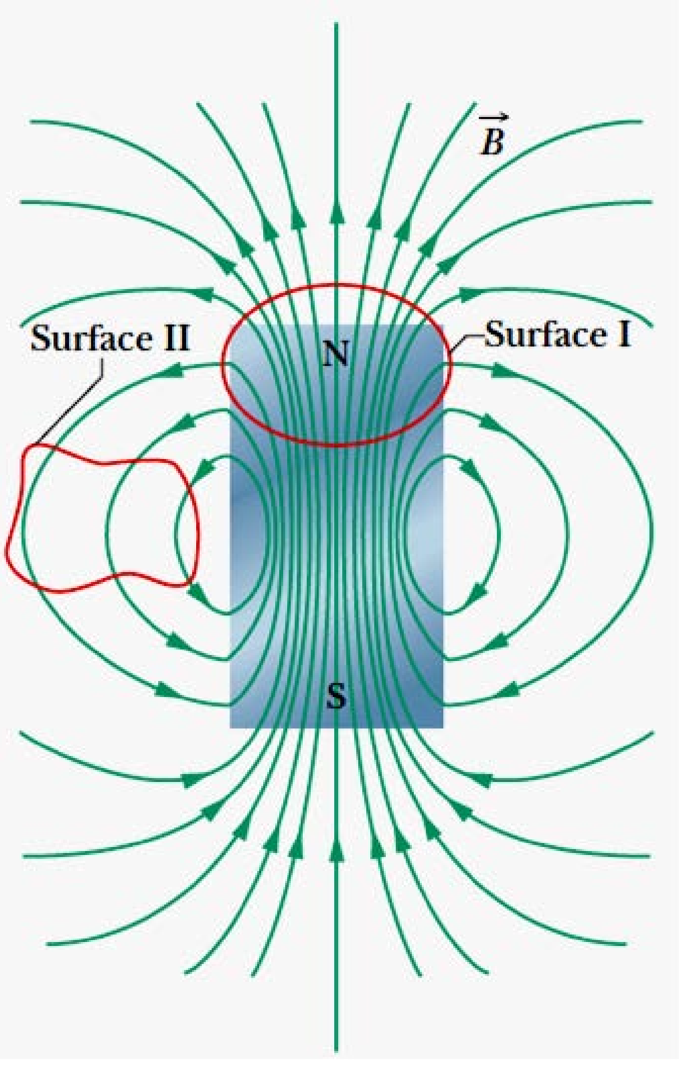


If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles

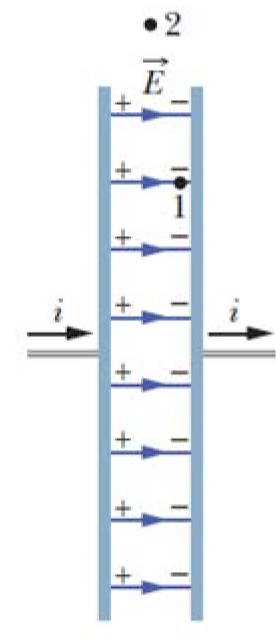
Gauss' law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure:

1. *Gaussian surface II* near the bar magnet of Fig. 32-4 encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

2. For *Gaussian surface I*, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. Thus, *Gaussian surface I* encloses a magnetic dipole, and the net flux through the surface is zero.



7.1.2. Induced Magnetic Fields:



(a)

We saw that a changing magnetic flux induces an electric field and we ended up with **Faraday's law of induction**:

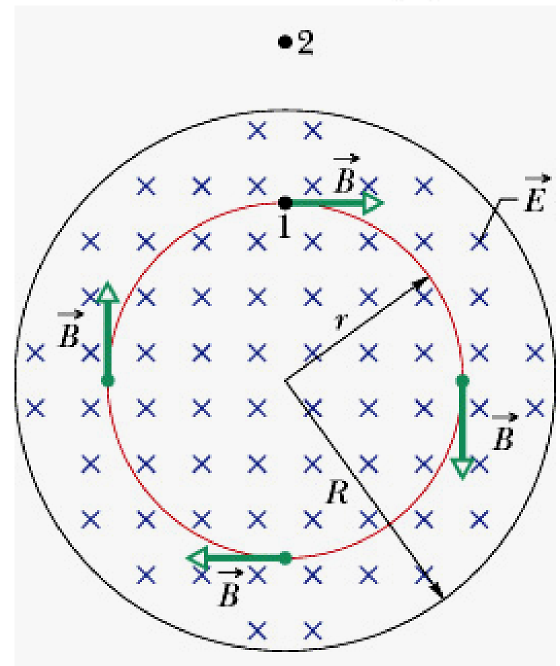
$$\oint \vec{E}d\vec{s} = -\frac{d\phi_B}{dt}$$

So, we should be tempted to ask: *Can a changing electric flux induce a magnetic field?* The answer: *It can*.

We have **Maxwell's law of induction**:

$$\oint \vec{B}d\vec{s} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$$

Here B is the magnetic field induced <u>along a closed loop</u> by the changing electric flux ϕ_E <u>in the region encircled by that loop</u>



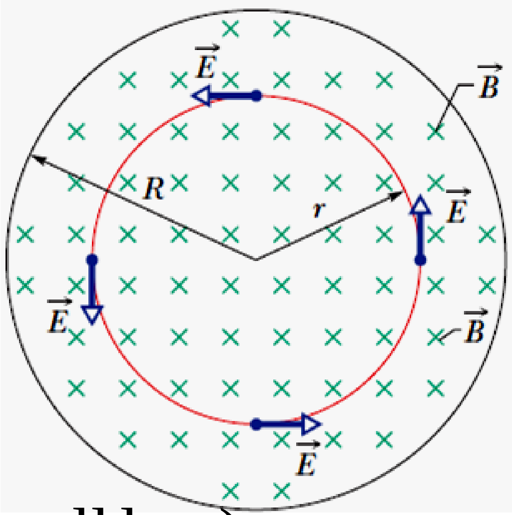
(b)

<u>Example:</u> A parallel-plate capacitor with circular plates is charging. We assume that the charge on the capacitor is being **increased** at a steady rate. So, the electric field magnitude between the plates is also increasing at a steady rate.

- Experiment proves that:
  - B is induced a loop directed as shown.

- B has the same magnitude at every point around the loop and thus has circular symmetry about the central axis of the plates.
- We also find that a magnetic field is induced around a larger loop as well, e.g. through point 2 outside the plates.
- The change of electric field induces B between the plates, both inside and outside the gap.

The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.



Ampere-Maxwell Law:

Recall Ampere's law:

$$\oint \vec{B} d\vec{s} = \mu_0 i_{enc} \text{ (Ampere's law)}$$

Here i_{enc} is the current encircled by the closed loop. In a more complete form:

$$\oint \vec{B} d\vec{s} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{enc} \text{ (Ampere - Maxwell law)}$$

•If there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of the second equation is zero, and so it reduces to the first equation, Ampere's law.

7.1.3. Displacement Current:

- In the Maxwell-Ampere equation, $\varepsilon(d\phi_E/dt)$ must have the dimension of a current. This product has been treated as being a fictitious current called the **displacement current** i_d:

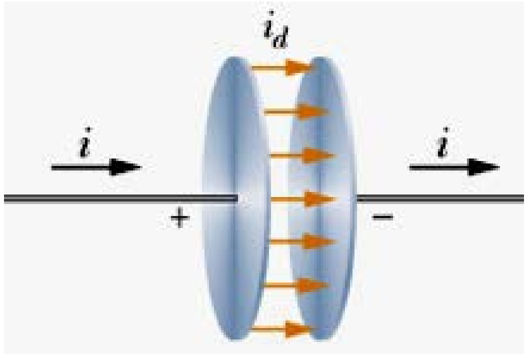
$$\boxed{i_d = \varepsilon_0 \frac{d\phi_E}{dt} \text{ (displacement current)}}$$

- So:

$$\oint \vec{B} d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc} \text{ (Ampere - Maxwell law)}$$

$i_{d,enc}$: the displacement current encircled by the integration loop.

- Consider a charging capacitor, the charge q on the plates: $$q = \varepsilon_0 AE$$



The real current i: $$\frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}$$

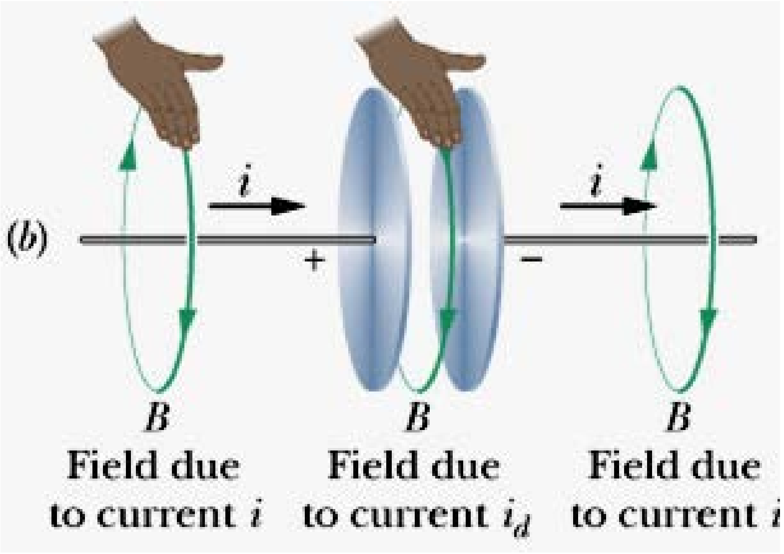
The displacement current i_d:

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = i$$

$$i_d = i \text{ (displacement current in a capacitor)}$$

• Thus, we can consider the fictitious current i_d to be simply a continuation of the real current i from one plate across the capacitor gap, to the other plate.

• Although **no charge** actually moves across the gap between the plates, the idea of the fictitious current i_d can help us to quickly find the direction and magnitude of an induced magnetic field:



Recall: Inside a long straight wire with current:

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

We consider the space between the plates to be an imaginary circular wire of radius R carrying the imaginary current i_d. So, B at a point inside the capacitor:

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r \text{ (inside a circular capacitor)}$$

At a point outside the capacitor: $$B = \frac{\mu_0 i_d}{2\pi r} \text{ (outside a circular capacitor)}$$

7.1.4. Maxwell's Equations (Integral form):

With the assumption that no dielectric or magnetic materials are present:

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} d\vec{A} = q_{enc} / \varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} d\vec{s} = -\frac{d\phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere-Maxwell law	$\oint \vec{B} d\vec{s} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{enc}$	Relates induced magnetic field to changing electric flux and to current

Maxwell's Equations (Differential form):

$$\boxed{\begin{aligned}
&\nabla \cdot E = \rho / \varepsilon_0 \\
&\nabla \cdot B = 0 \\
&\nabla \times E = -\frac{\partial B}{\partial t} \\
&\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 j_c \\
&\text{where } \nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}
\end{aligned}}$$

ρ: charge density; ε_0: permittivity;
μ_0: permeability; j_c: current density

∇: del or nabla operator
∇•: the divergence operator
∇×: the curl operator

For example, a vector function F(x,y,z):

$$\mathrm{div}F = \nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\mathrm{curl}F = \nabla \times F = \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

or

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

7.2. Magnetism of Matter:

7.2.1. Magnets: The Magnetism of Earth:

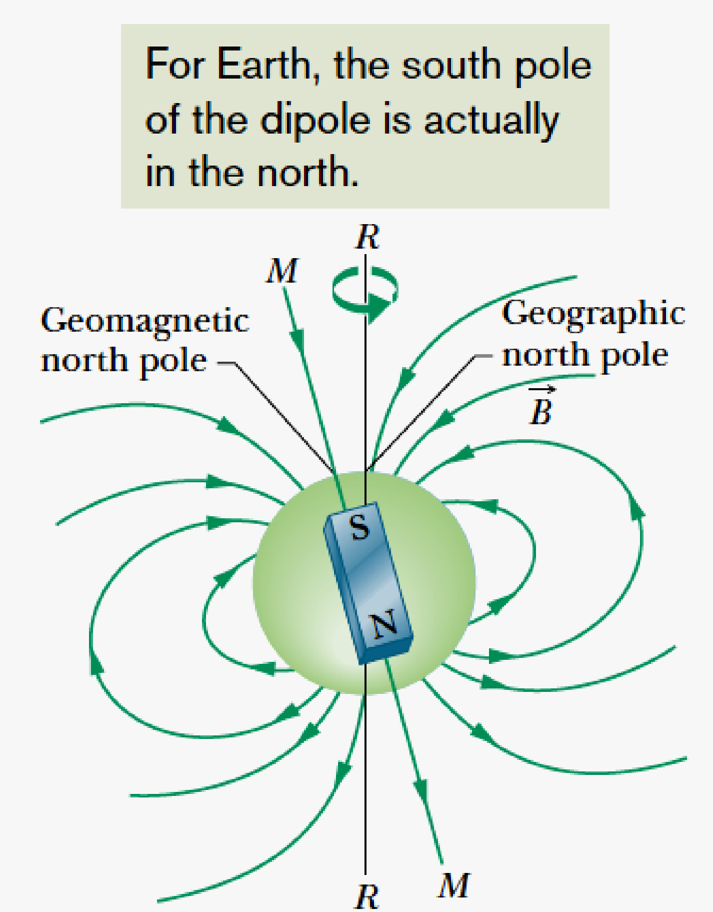


Fig. 32-8 Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

- *A magnet is a material or object that produces a magnetic field.*

- Earth is a huge magnet, its magnetic field can be approximated as the field of a huge bar magnet – a magnetic dipole – that straddles the center of the planet.

- The magnitude of Earth's magnetic field at the Earth's surface ranges from 25 to 65 μT (0.25-0.65 G).

- At any point on Earth's surface, the measured magnetic field may differ appreciably, in both magnitude and direction, from the idealized dipole field.

7.2.2. Magnetism and Electrons:

• *Magnetic materials, from lodestones to videotapes, are magnetic because of the electrons within them.*

• We have already seen one way in which electrons generate a magnetic field: send them through a wire as an electric current, and their motion produces a magnetic field around the wire.

• There are two more ways, each involving a magnetic dipole moment that produces a magnetic field in the surrounding space.

Spin Magnetic Dipole Moment:

An electron has an intrinsic angular momentum called its spin angular momentum (or just spin), ***S***; associated with this spin is an intrinsic spin magnetic dipole moment, μ_s .

$$\vec{\mu}_S = -\frac{e}{m}\vec{S}$$

in which *e* is the elementary charge (1.60×10^{-19} *C*) and *m* is the mass of an electron (9.11×10^{-31} kg).

Spin $\vec{S}$ is different from the angular momentum $\vec{l} = \vec{r} \times \vec{p}$ in two respects:

• Spin $\vec{S}$ itself cannot be measured. However, its component along any axis can be measured.
• A measured component of $\vec{S}$ is quantized, which is a general term that means it is restricted to certain values. A measured component of $\vec{S}$ can have only two values, which differ only in sign.

Assuming that the component of spin S is measured along the z axis of a coordinate system:

$$S_z = m_S \frac{h}{2\pi}, \text{for } m_S = \pm\frac{1}{2}$$

where m_S is called the *spin magnetic quantum number* and h=6.63 $\times$ 10^{-34} Js is the Planck constant.

- When S_z is parallel to the z axis, m_S is +1/2 and the electron is said to be *spin up*.
- When S_z is antiparallel to the z axis,m_S is -1/2 and the electron is said to be *spin down*.

The spin magnetic dipole moment $\vec{\mu}_S$ of an electron also cannot be measured, only its component along any axis can be measured. and that component is quantized too.

$$\mu_{S,z} = -\frac{e}{m} S_z$$

$$\Rightarrow \mu_{S,z} = \pm \frac{eh}{4\pi m}$$

"+" and "-" correspond to $\mu_{S,z}$ being parallel and antiparallel to the z axis, respectively.

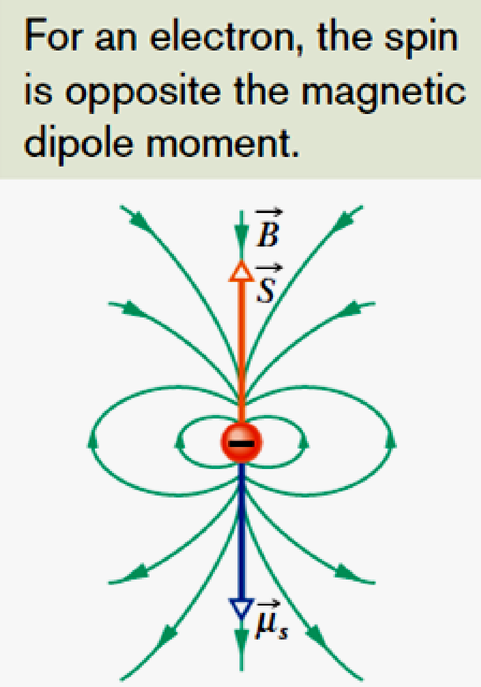


Fig. 32-10 The spin $\vec{S}$, spin magnetic dipole moment $\vec{\mu}_s$, and magnetic dipole field $\vec{B}$ of an electron represented as a microscopic sphere.

$$\boxed{\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ J/T (Bohr magneton)}}$$

When an electron in an external B field, a potential energy U can be associated with the orientation of the electron's $\vec{\mu}_S$ when B_{ext} is the exterior magnetic field aligned along the z-axis.

$$U = -\vec{\mu}_S \cdot \vec{B}_{ext} = -\mu_{S,z} B_{ext}$$

Note: We do not examine the contribution of the magnetic dipole moments of protons and neutrons to the magnetic field of atoms because they are about a thousand times smaller than that due to an electron, due to their much larger mass.

Orbital Magnetic Dipole Moment:

• When an electron is in an atom, it has an additional angular momentum called its orbital angular momentum, $\vec{L}_{\text{orb}}$. Associated with it is an orbital magnetic dipole moment, $\vec{\mu}_{\text{orb}}$; the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}$$

• Only the component along any axis of the orbital angular momentum can be measured, and that component is quantized

$$L_{\text{orb,z}} = m_l \frac{h}{2\pi} \text{ for } m_l = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit})$$

in which is m_l called the orbital magnetic quantum number and "limit" refers to its largest allowed integer value.

• Similarly, only the component of the magnetic dipole moment of an electron along an axis can be measured, and that component is quantized.

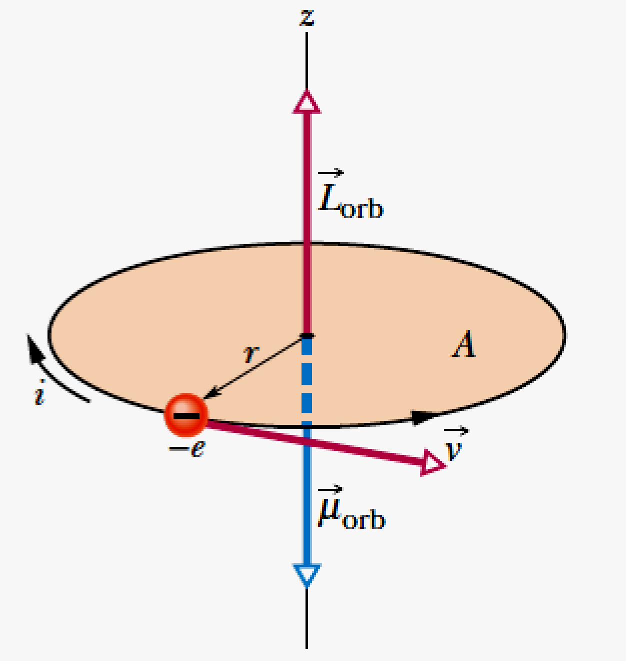
$$\Rightarrow \mu_{\text{orb},z} = -m_l \frac{eh}{4\pi m} = -m_l \mu_B \; (\mu_B : \text{Bohr magneton})$$

• In an external B field, a potential energy U:

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb,z}} B_{\text{ext}}$$

where the z axis is taken in the direction of B_{ext}

Loop Model for Electron Orbits:



We can obtain the relationship between μ_{orb} and L_{orb} with the non-quantum derivation as follows:

- We imagine an electron moving in a circular path as shown
- The motion of the electron is equivalent to a current i
- The magnitude of the orbital magnetic dipole moment of such a current loop is:

$$\mu_{orb} = iA$$

where A is the area enclosed by the loop

The direction of $\vec{\mu}_{orb}$ is downward

The current:

$$i = \frac{\text{charge}}{\text{time}} = \frac{e}{2\pi r / v} \Rightarrow \mu_{orb} = \frac{e}{2\pi r / v}\pi r^2 = \frac{evr}{2} (1)$$

The electron's orbital angular momentum (using $\vec{l} = \vec{r} \times \vec{p}$):

$$L_{orb} = mrv \sin 90^0 = mrv \ (2)$$

The direction of $\vec{L}_{orb}$ is upward

(1) & (2):

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

Loop Model for Electron Orbits in a Nonuniform Field:

• We study an electron orbit as a current loop but now in a nonuniform magnetic field. This helps us understand forces acting in magnetic materials in a nonuniform magnetic field.

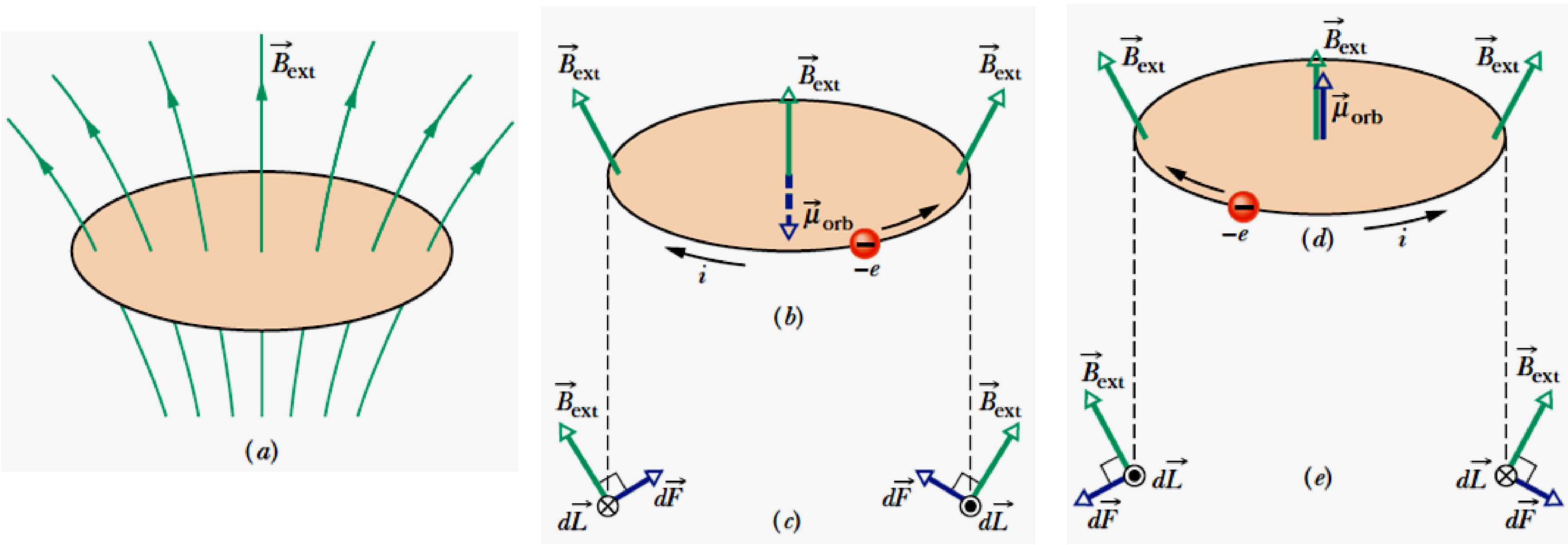


Fig. 32-12 (a) A loop model for an electron orbiting in an atom while in a nonuniform magnetic field $\mathbf{B}_{ext}$. (b) Charge e moves counterclockwise; the associated conventional current *i* is clockwise. (c) The magnetic forces ***dF*** on the left and right sides of the loop, as seen from the plane of the loop. The net force on the loop is upward. (d) Charge e now moves clockwise. (e) The net force on the loop is now downward.

$$d\vec{F} = i d\vec{L} \times \vec{B}_{ext}$$

7.2.3. Magnetic Materials:

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment. In a magnetic material the combination of all the magnetic dipole moments produces a magnetic field. There are three general types of magnetism:

1. **Diamagnetism:** In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\mathbf{B}_{ext}$; the combination gives the material as a whole feeble net magnetic field.
2. **Paramagnetism:** Each atom of such a material has a permanent resultant magnetic dipole moment, but the moments are randomly oriented in the material and the material lacks a net magnetic field. An external magnetic field $\mathbf{B}_{ext}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field.
3. **Ferromagnetism:** Some of the electrons in these materials have their resultant magnetic dipole moments aligned, which produces regions with strong magnetic dipole moments. An external field $\mathbf{B}_{ext}$ can align the magnetic moments of such regions, producing a strong magnetic field for the material.

Diamagnetism:

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

If a magnetic field is applied, the diamagnetic material develops a magnetic dipole moment and experiences a magnetic force. When the field is removed, both the dipole moment and the force disappear.

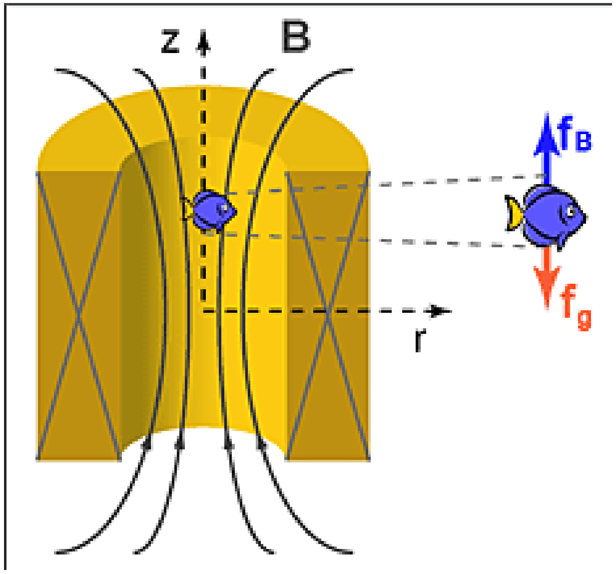
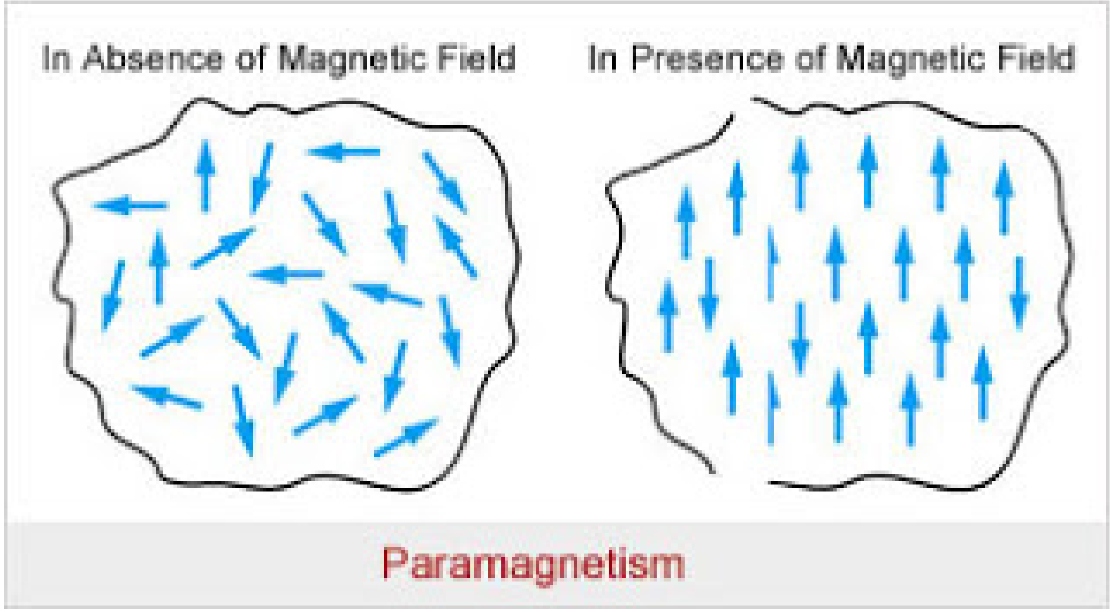


Fig. 32-13 An overhead view of a frog that is being levitated in a magnetic field produced by current in a vertical solenoid below the frog. *(Courtesy A. K. Gein, High Field Magnet Laboratory, University of Nijmegen, The Netherlands)*

Paramagnetism:

A paramagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the paramagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

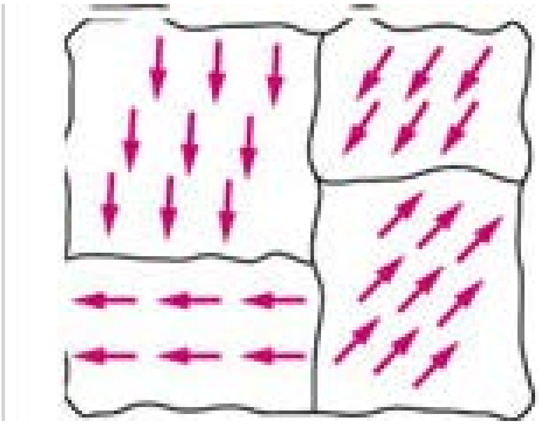


Liquid oxygen is suspended between the two pole faces of a magnet because the liquid is paramagnetic and is magnetically attracted to the magnet. *(Richard Megna/Fundamental Photographs)*

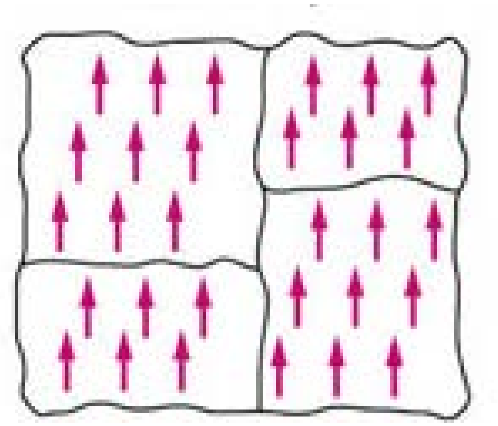
Ferromagnetism:

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

In Absence of Magnetic Field



In Presence of Magnetic Field



- Ferromagnetism is very important in industry and modern technology, and is the basis for many electrical and electromechanical devices such as electric motors, generators, transformers, and magnetic storage such as hard disks.

- Every ferromagnetic substance has its own individual temperature, called the Curie temperature, or Curie point, above which it loses its ferromagnetic properties.

• The magnetization of a ferromagnetic material such as iron can be studied with an arrangement called a *Rowland ring (Fig. 32-15).*

• The material is formed into a thin toroidal core of circular cross section. A primary coil P having *n turns* per unit length is wrapped around the core and carries current i_P. If the iron core were not present, the magnitude of the magnetic field inside the coil would be:

$$B_0 = \mu_0 i_P n$$

• With the iron core present, the magnetic field inside the coil is greater than $\mathbf{B}_0$, usually by a large amount:

$$B = B_0 + B_M$$

Here $\boldsymbol{B}_M$ is the magnitude of the magnetic field contributed by the iron core.

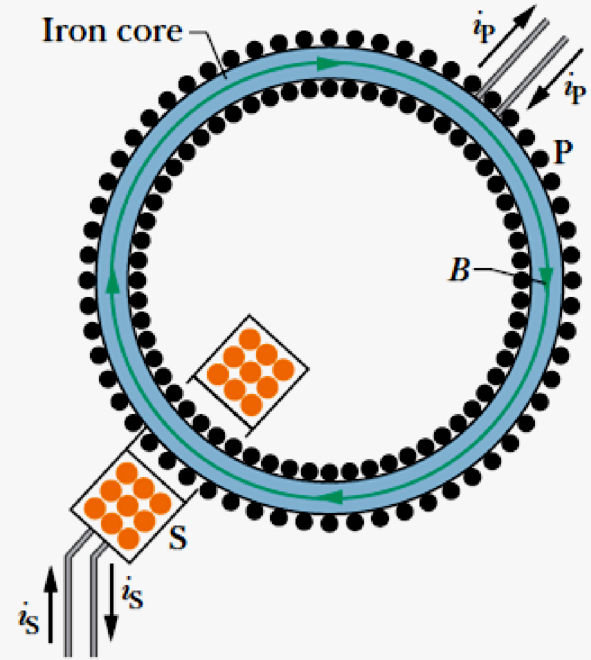


Fig. 32-15 A Rowland ring. A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field $\vec{B}$ within coil P. Field $\vec{B}$ can be measured by means of a secondary coil S.

Hysteresis

Magnetization curves for ferromagnetic materials are not retraced as we increase and then decrease the external magnetic field B_0.

Figure 32-18 is a plot of B_M versus B_0 during the following operations with a Rowland ring:

I. Starting with the iron unmagnetized (point *a*), increase the current in the toroid until B_0 $(=m_0 in)$;

II. Reduce the current in the toroid winding (and thus B_0) back to zero (*c*);

III. Reverse the toroid current and increase it in magnitude until B_0 has the value corresponding to point *d*;

IV. Reduce the current to zero again (*e*);

V. Reverse the current once more until point *b* is reached again.

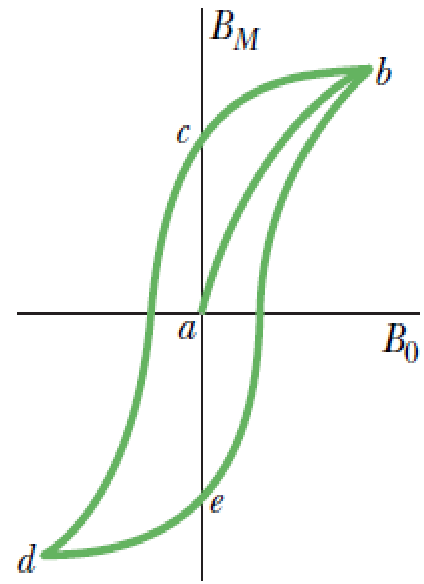


Fig. 32-18 A magnetization curve (*ab*) for a ferromagnetic specimen and an associated hysteresis loop (*bcdeb*).

The lack of retraceability shown in Fig. 32-18 is called **hysteresis**, and the curve *bcdeb* is called a ***hysteresis loop***.

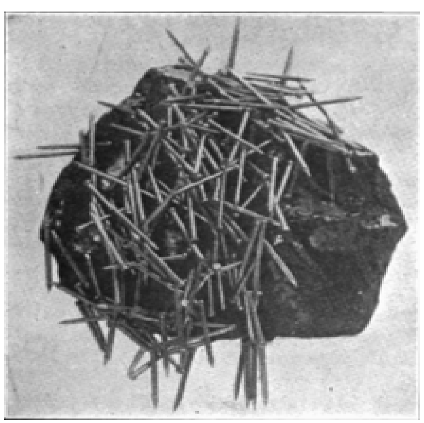
• Hysteresis can be understood through the concept of magnetic domains, which are regions of a ferromagnetic material in which the magnetic dipole moments are aligned parallel:

- When the applied B_0 is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some "memory" of their alignment. This memory of magnetic materials is applicable for the magnetic storage of information, as on magnetic tapes and disks.

(image source: wikipedia)

- This memory of the alignment of domains can also occur naturally. What is the origin of lodestones?

Lightning sends currents through the ground that produce strong magnetic fields. The fields can suddenly magnetize any ferromagnetic material in nearby rock. Because of hysteresis, such rock material retains some of that magnetization after the lightning strike.



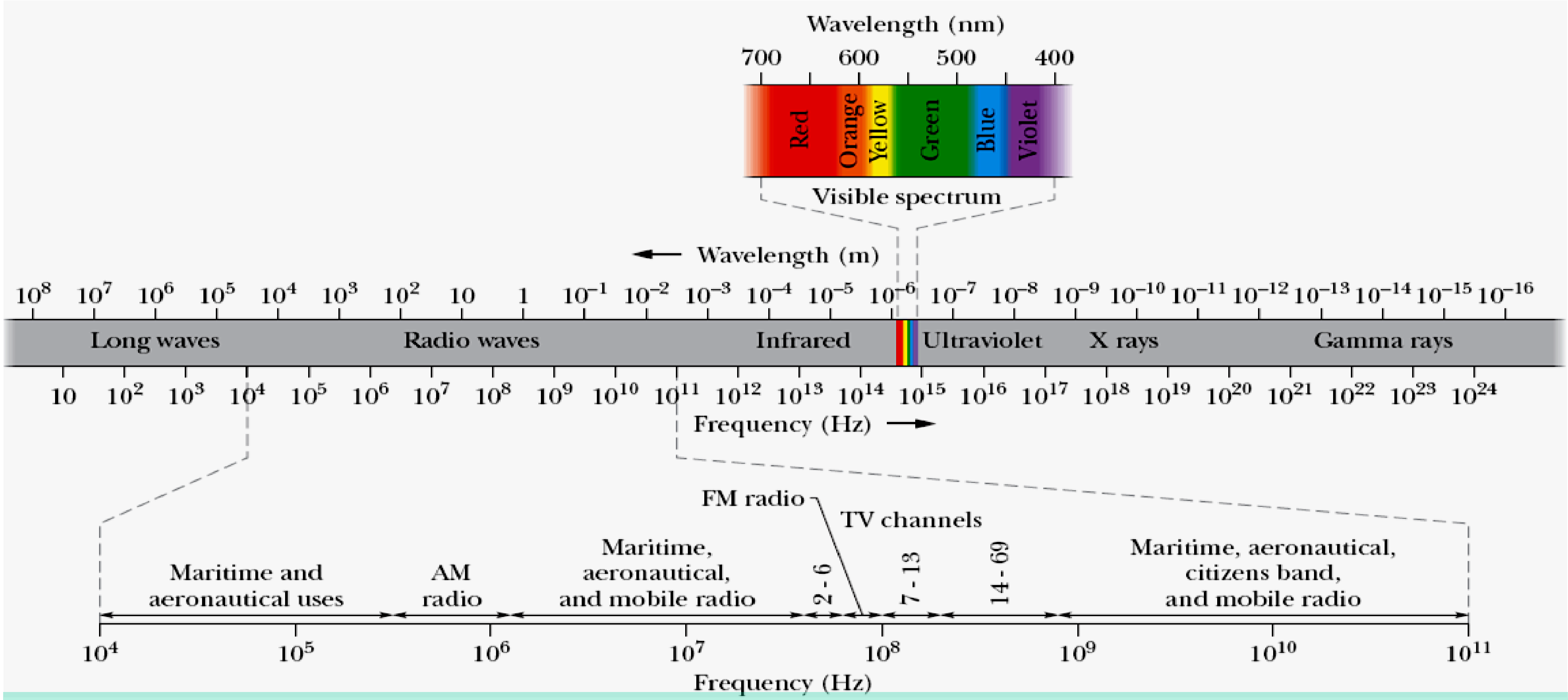
Overview

- The information age in which we live is based almost entirely on the physics of electromagnetic waves.
- Like it or not, we are now globally connected by TV, telephones, and the Web.
- Like it or not, we are constantly immersed in those signals because of TV, radio, and telephone transmitters.
- The challenge for today's engineers is trying to envision what the global interconnection will be like 20 years from now.
- The starting point in meeting that challenge is understanding the basic physics of electromagnetic waves

7.3. Electromagnetic waves:

7.3.1. Maxwell's Rainbow:

- An electromagnetic wave consists of oscillating electric and magnetic fields.
- The various possible frequencies of electromagnetic waves form a *spectrum*. It is poetically called Maxwell's rainbow.



The scale is open-ended; the wavelengths/frequencies of electromagnetic waves have no inherent upper or lower bound.

- **A small part if this spectrum is visible light.**

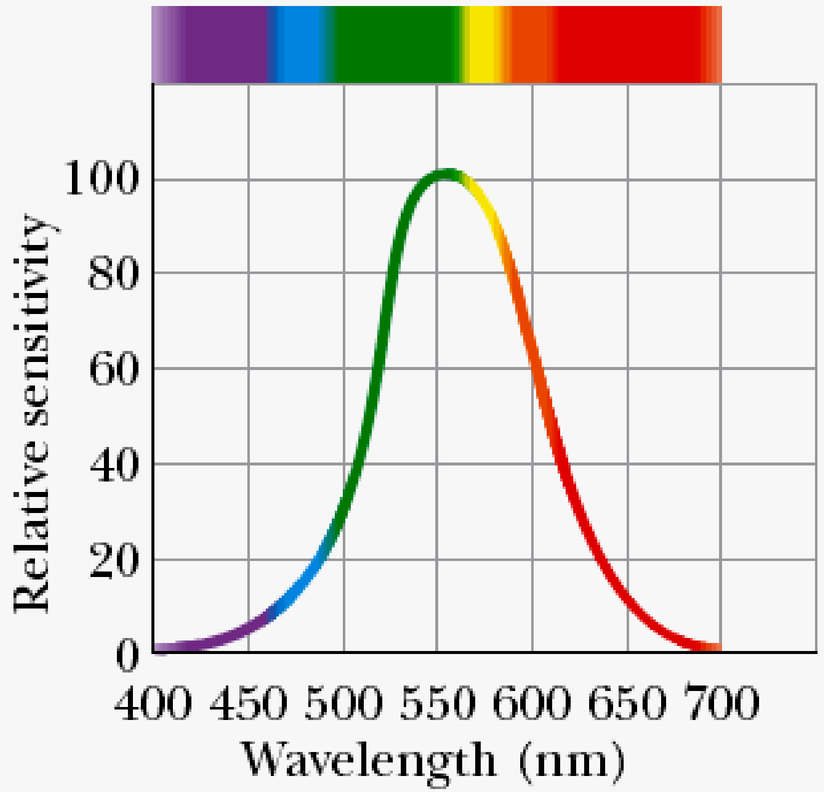


Fig. 33-2 The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called *visible light*.

7.3.2. The Traveling Wave, Qualitatively:

• Some electromagnetic waves, including x rays, gamma rays, and visible light, are *radiated (emitted)* from sources of atomic or nuclear size.

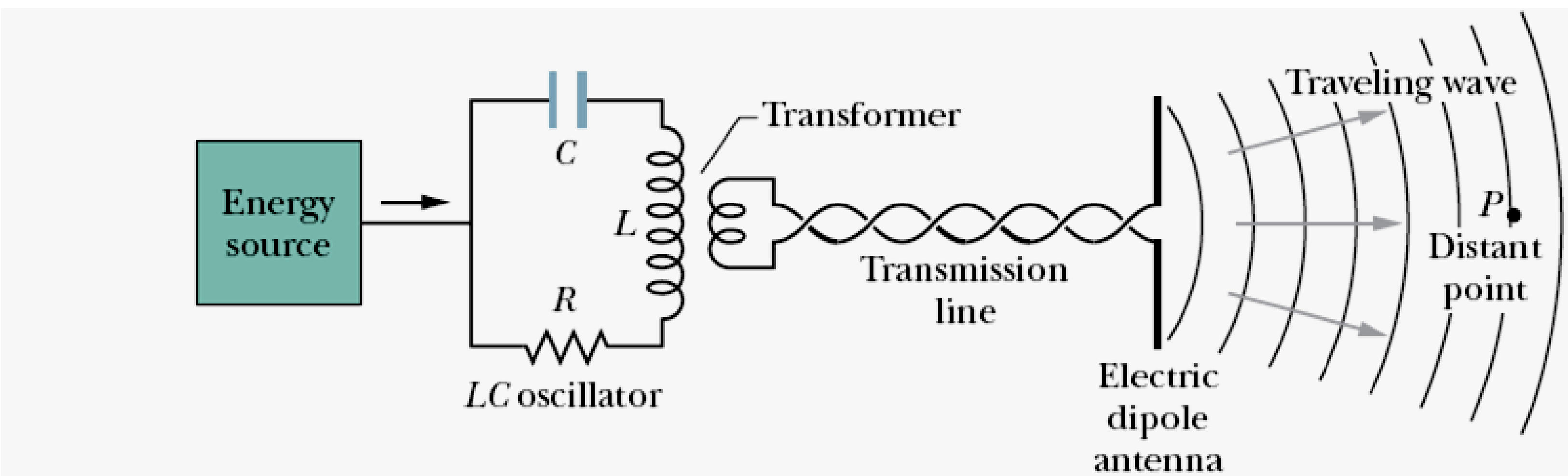
• How other electromagnetic waves are generated?

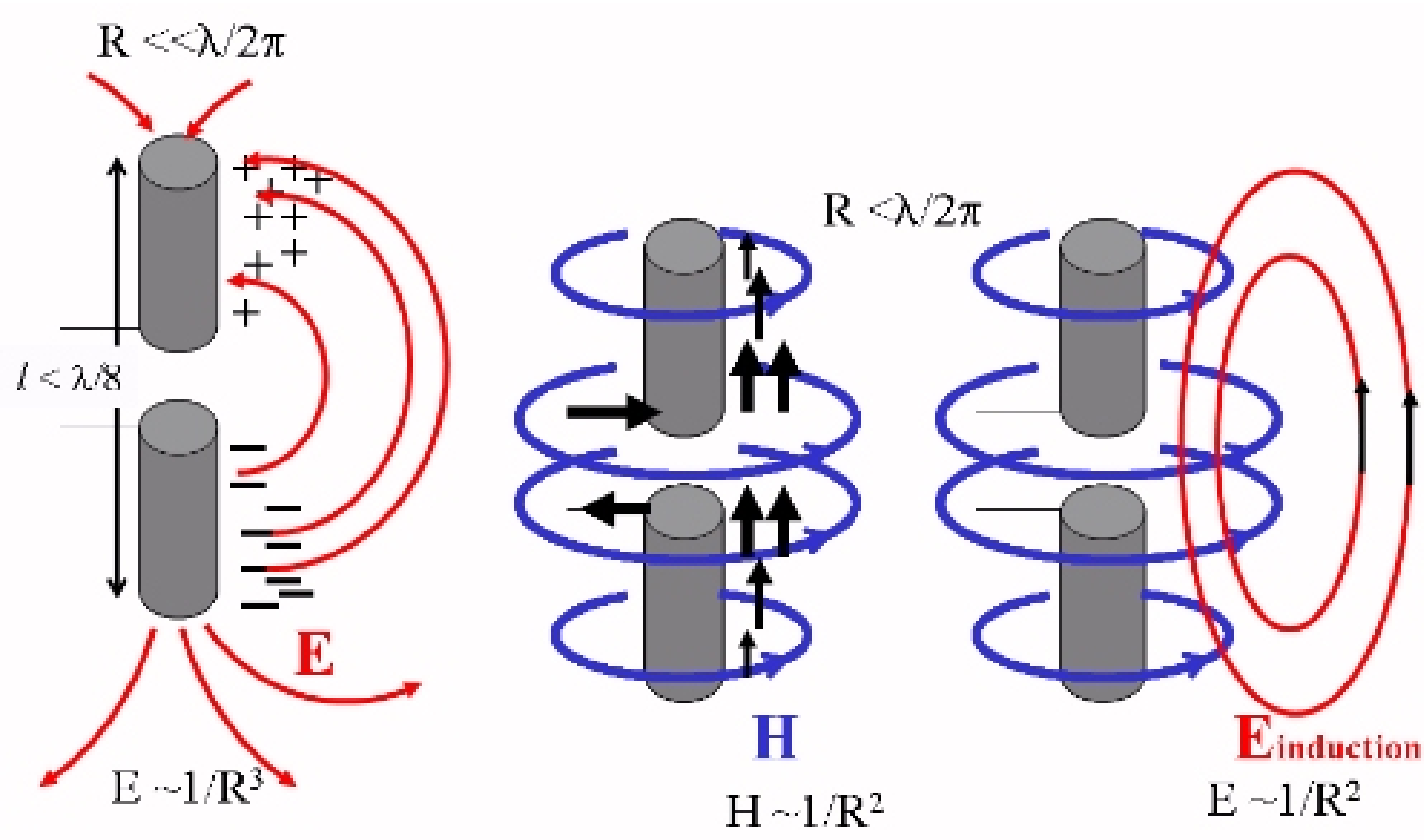
To simply matters we only study the source of the radiation that is both macroscopic and of manageable dimensions:

✓ The figure shows the generation of such waves. At its heart is an *LC oscillator*, which establishes an angular frequency $\omega = 1/\sqrt{LC}$

✓ Charges and currents in this circuit vary sinusoidally at this frequency.

✓ An energy source –possibly an ac generator- must be included to supply energy to compensate both for thermal losses and for energy carried away by the radiated electromagnetic wave.





R <<λ/2π
l < λ/8
E
E ~1/R³
R <λ/2π
H
H ~1/R²
Einduction
E ~1/R²

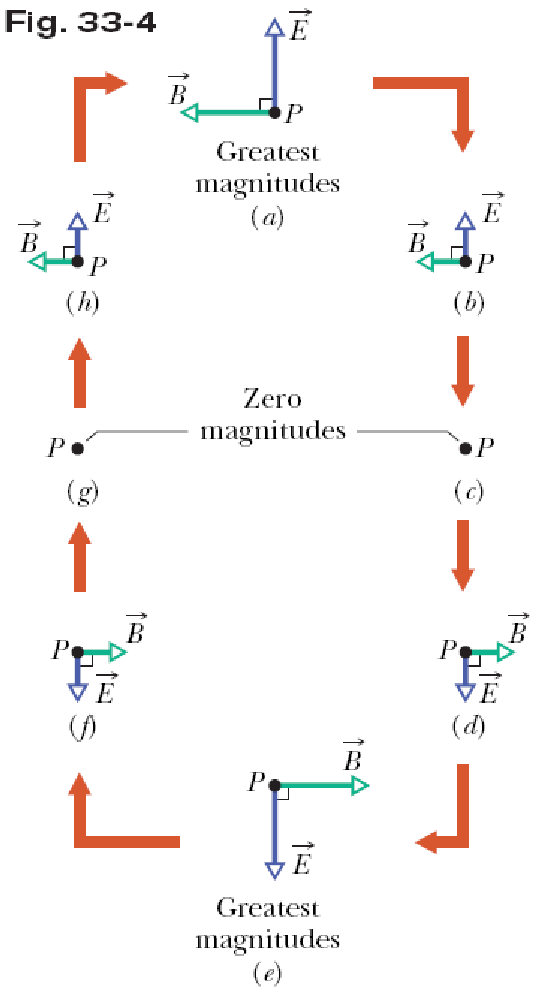


Figure 33-4 shows how the electric field and the magnetic field change with time as one wavelength of the wave sweeps past the distant point *P* in the last figure; in each part of Fig. 33-4, the wave is traveling directly out of the page.

At a distant point, P, the curvature of the waves is small enough to neglect it. At such points, the wave is said to be a ***plane wave***.

Key features regardless of how the waves are generated:

1. The electric and magnetic fields and are always perpendicular to the direction in which the wave is traveling. The wave is a ***transverse wave***.

2. The electric field is always perpendicular to the magnetic field.

3. The cross product always gives the direction in which the wave travels.

4. The fields always vary sinusoidally. The fields vary with the same frequency and are ***in phase*** with each other.

We can write the electric and magnetic fields as sinusoidal functions of position x (along the path of the wave) and time t :

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

Here E_m and B_m are the amplitudes of the fields and, ω and k are the angular frequency and angular wave number of the wave, respectively.

All electromagnetic waves, including visible light, have the same speed c in vacuum.

The speed of the wave (in vacuum) is given by c.

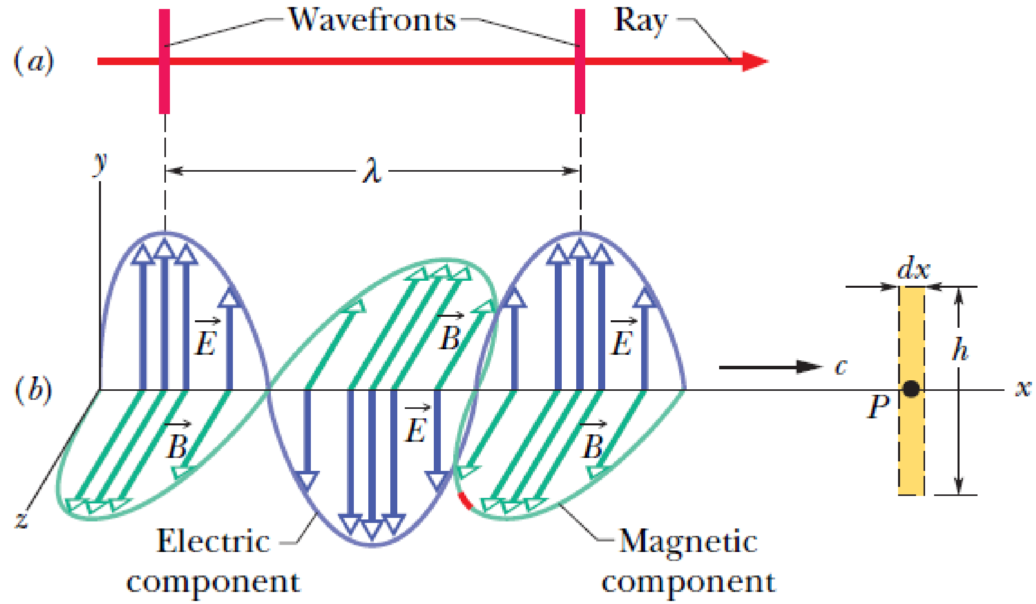
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(wave speed)},$$

Its value is about 3.0 x10^8 m/s.

Quantitatively:

The dashed rectangle of dimensions *dx* and *h* in Fig. 33-6 is fixed at point *P*.

As the electromagnetic wave moves rightward past the rectangle, the magnetic flux *B* through the rectangle changes → the induced electric fields appear throughout the region of the rectangle. We take **E** and **E+dE** to be the induced fields along the two long sides of the rectangle. These induced electric fields are the electrical component of the electromagnetic wave.



$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} = (E + dE)h - Eh = h\,dE.$$

$$\frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt} \Rightarrow h\,dE = -h\,dx\,\frac{dB}{dt} \Rightarrow \frac{dE}{dx} = -\frac{dB}{dt}.$$

$$\frac{\partial E}{\partial x} = kE_m\cos(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = -\omega B_m\cos(kx - \omega t).$$

$$kE_m\cos(kx - \omega t) = \omega B_m\cos(kx - \omega t).$$

$$\boxed{\frac{E_m}{B_m} = c \qquad \text{(amplitude ratio),}}$$

The oscillating electric field induces an oscillating and perpendicular magnetic field.

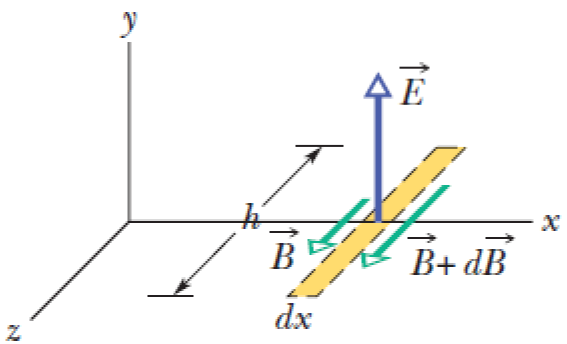


Fig. 33-7 The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point *P* in *Fig. 33-5b*, ***E*** induces magnetic fields along the rectangle.The instant shown is that of Fig. 33-6: is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt},$$

$$\oint \vec{B}\cdot d\vec{s} = -(B + dB)h + Bh = -h\,dB.$$

$$\Phi_E = (E)(h\,dx), \qquad \frac{d\Phi_E}{dt} = h\,dx\,\frac{dE}{dt}$$

$$-h\,dB = \mu_0\varepsilon_0\left(h\,dx\,\frac{dE}{dt}\right)$$

$$-\frac{\partial B}{\partial x} = \mu_0\varepsilon_0\frac{\partial E}{\partial t}.$$

$$-kB_m\cos(kx-\omega t) = -\mu_0\varepsilon_0\omega E_m\cos(kx-\omega t),$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0\varepsilon_0(\omega/k)} = \frac{1}{\mu_0\varepsilon_0 c}.$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \text{(wave speed)},}$$

7.4. Energy Transport and the Poynting Vector:

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector).}$$

$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}.$$

$$S = \frac{1}{\mu_0}EB, \quad \Rightarrow \quad S = \frac{1}{c\mu_0}E^2$$

$$I = S_{\text{avg}} = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{avg}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{avg}} = \frac{1}{c\mu_0}[E^2]_{\text{avg}} = \frac{1}{c\mu_0}[E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.$$

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \quad \Rightarrow \quad I = \frac{1}{c\mu_0}E_{\text{rms}}^2.$$

The energy density u (= ½ $\varepsilon_0 E^2$) within an electric field, can be written as:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0 (cB)^2. = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0\varepsilon_0}B^2 = \frac{B^2}{2\mu_0}$$

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}$$

The energy emitted by light source S must pass through the sphere of radius r.

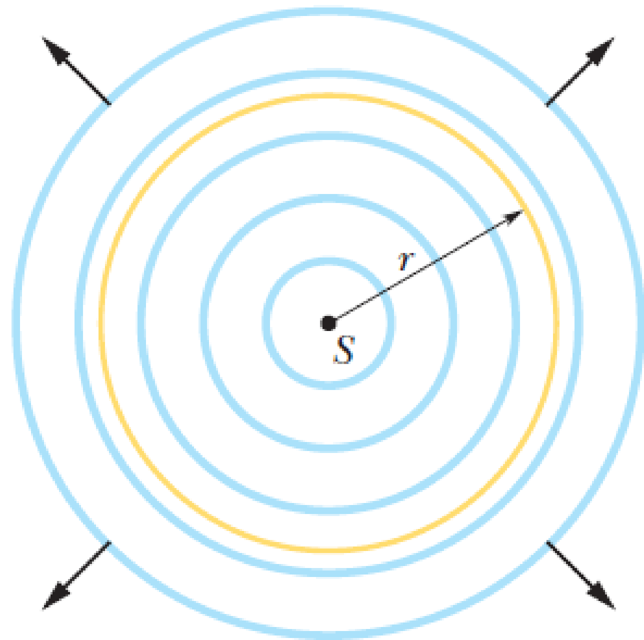


Fig. 33-8 A point source S emits electromagnetic waves uniformly in all directions. The spherical wavefronts pass through an imaginary sphere of radius r that is centered on S.

7.5. Radiation Pressure:

Electromagnetic waves have linear momentum and thus can exert a pressure on an object when shining on it.

During the interval Δt, *the object gains* an energy ΔU from the radiation. If the object is free to move and that the radiation is entirely **absorbed** (taken up) by the object, then the momentum change Δp is given by:

$$\Delta p = \frac{\Delta U}{c} \qquad \text{(total absorption)},$$

If the radiation is entirely reflected back along its original path, the magnitude of the momentum change of the object is twice that given above, or

$$\Delta p = \frac{2\,\Delta U}{c} \qquad \text{(total reflection back along path)}.$$

Since $F = \dfrac{\Delta p}{\Delta t}$. and $I = \dfrac{\text{power}}{\text{area}} = \dfrac{\text{energy/time}}{\text{area}}$ it follows that:

$$F = \frac{IA}{c} \qquad \text{(total absorption)}. \qquad \text{AND} \qquad F = \frac{2IA}{c} \qquad \text{(total reflection back along path)}.$$

Finally, the radiation pressure in the two cases are:

$$p_r = \frac{I}{c} \qquad \text{(total absorption)}$$

$$p_r = \frac{2I}{c} \qquad \text{(total reflection back along path)}.$$

7.6. Polarization, Reflection and Refraction:

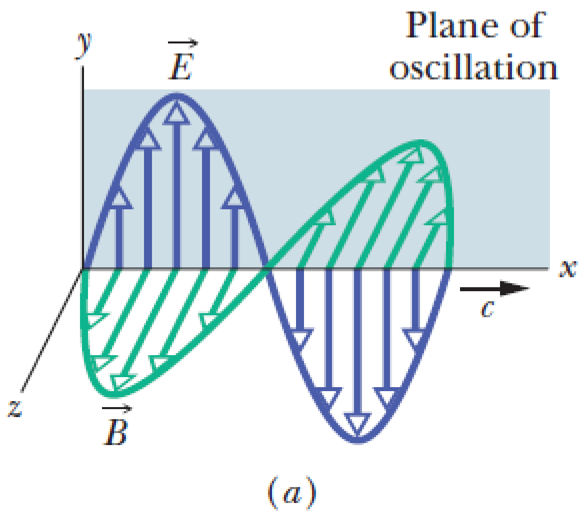
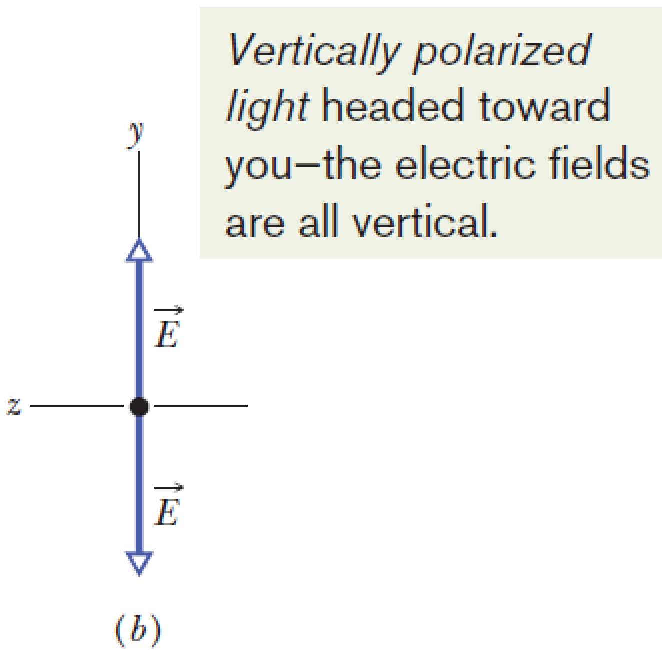


Fig. 33-9 (a) The plane of oscillation of a polarized electromagnetic wave. (b) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.



7.6.1. Polarization:

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

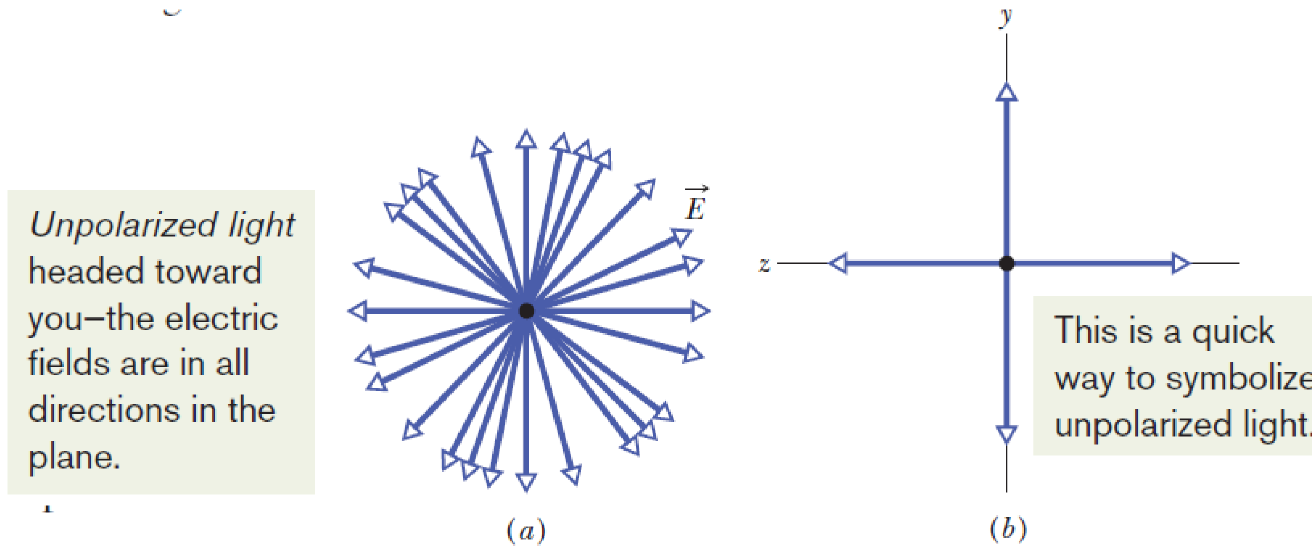


Fig. 33-10 (*a*) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude *E*. (*b*) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

If the intensity of original unpolarized light is I_o, then the intensity of the emerging light through the polarizer, I, is half of that.

$$I = \tfrac{1}{2} I_0.$$

Intensity of Polarized Light:

Suppose now that the light reaching a polarizing sheet is already polarized.

Figure 33-12 shows a polarizing sheet in the plane of the page and the electric field of such a polarized light wave traveling toward the sheet (and thus prior to any absorption).

We can resolve ***E*** into two components relative to the polarizing direction of the sheet: parallel component E_y is transmitted by the sheet, and perpendicular component E_z is absorbed. Since θ is the angle between and the polarizing direction of the sheet, the transmitted parallel component is

$$E_y = E \cos \theta.$$

Since $$I = E_{\rm rms}^2/c\mu_0$$

$$\boxed{I = I_0 \cos^2 \theta.}$$

The sheet's polarizing axis is vertical, so only vertical components of the electric fields pass.

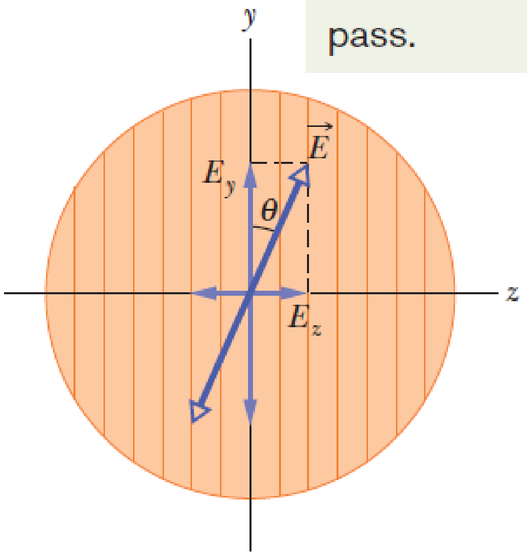


Fig. 33-12 Polarized light approaching a polarizing sheet. The electric field $\vec{E}$ of the light can be resolved into components E_y (parallel to the polarizing direction of the sheet) and E_z (perpendicular to that direction). Component E_y will be transmitted by the sheet; component E_z will be absorbed.

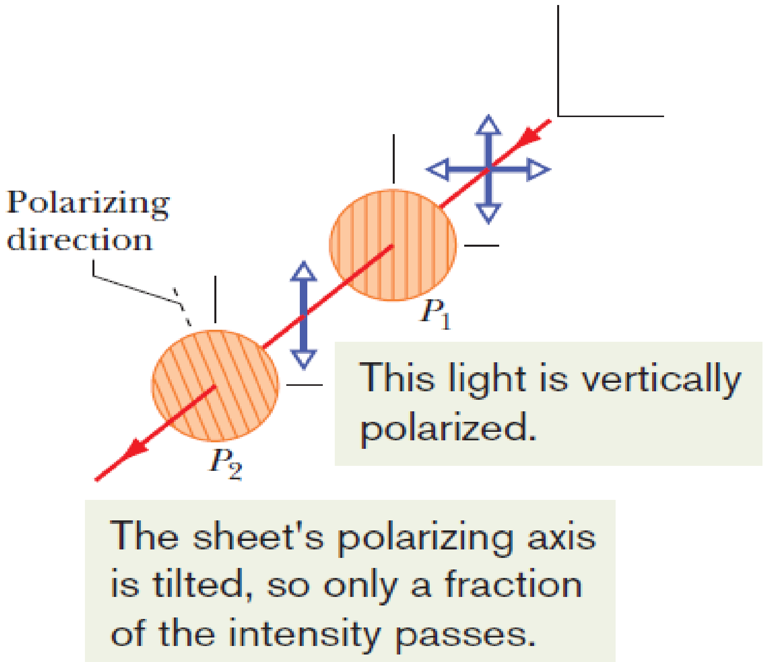


Fig. 33-13 The light transmitted by polarizing sheet P_1 is vertically polarized, as represented by the vertical double arrow. The amount of that light that is then transmitted by polarizing sheet P_2 depends on the angle between the polarization direction of that light and the polarizing direction of P_2 (indicated by the lines drawn in the sheet and by the dashed line).

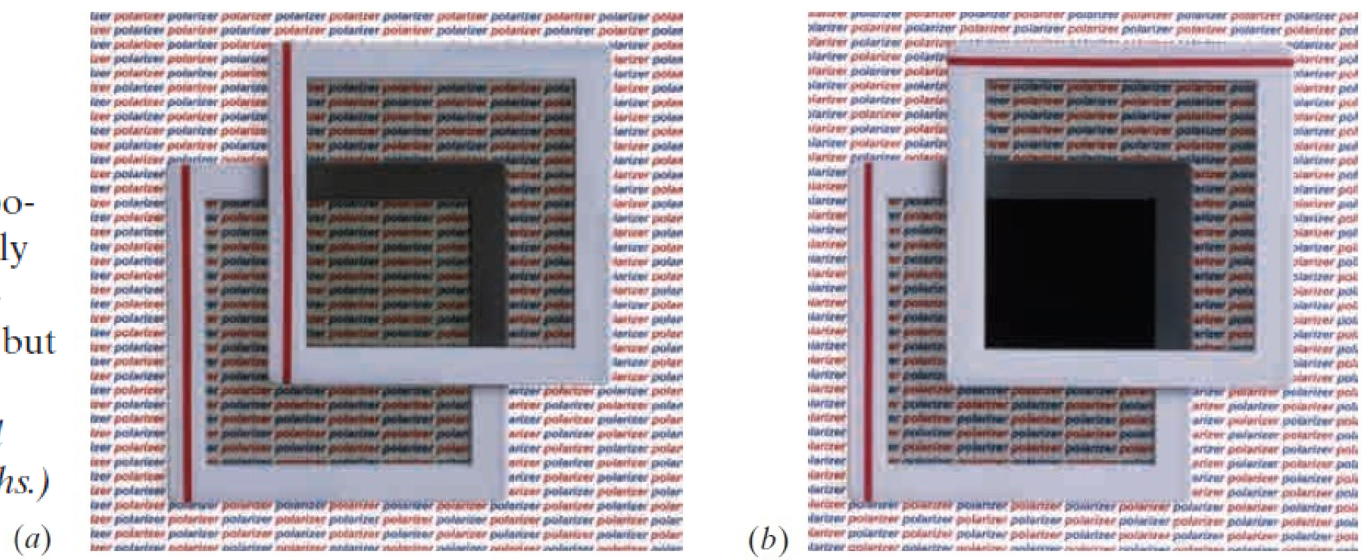


Fig. 33-14 (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed. *(Richard Megna/Fundamental Photographs.)*

7.6.2. Reflection and Refraction:

$$\theta_1' = \theta_1 \qquad \text{(reflection).}$$

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \qquad \text{(refraction).}$$

The index of refraction, n, of a medium is equal to c/v, where v is the speed of light in that medium and c is its speed in vacuum. In the refraction law, each of the symbols n_1 and n_2 is a dimensionless constant, called the **index of refraction,** that is associated with a medium involved in the refraction**.** The refraction law is also called **Snell's law**.

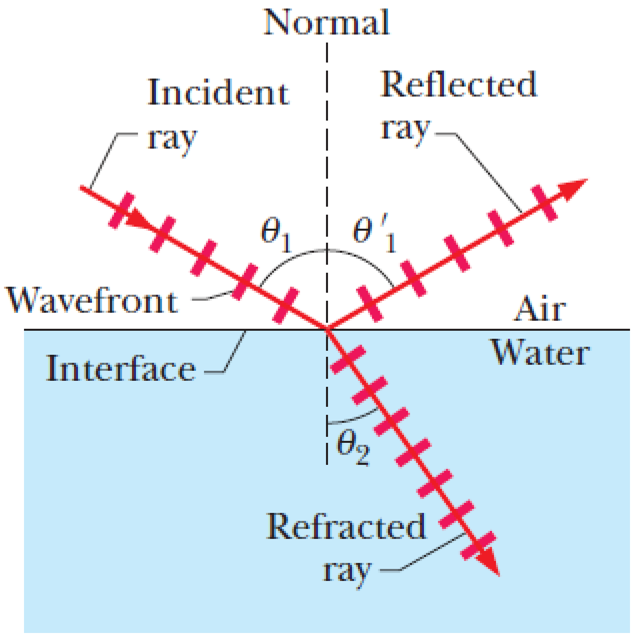


Fig. 33-16 (Continued) (*b*) A ray representation of (*a*). The angles of incidence (θ_1), reflection (θ_1'), and refraction (θ_2) are marked.

Table 33-1

Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).

[b]STP means "standard temperature (0°C) and pressure (1 atm)."

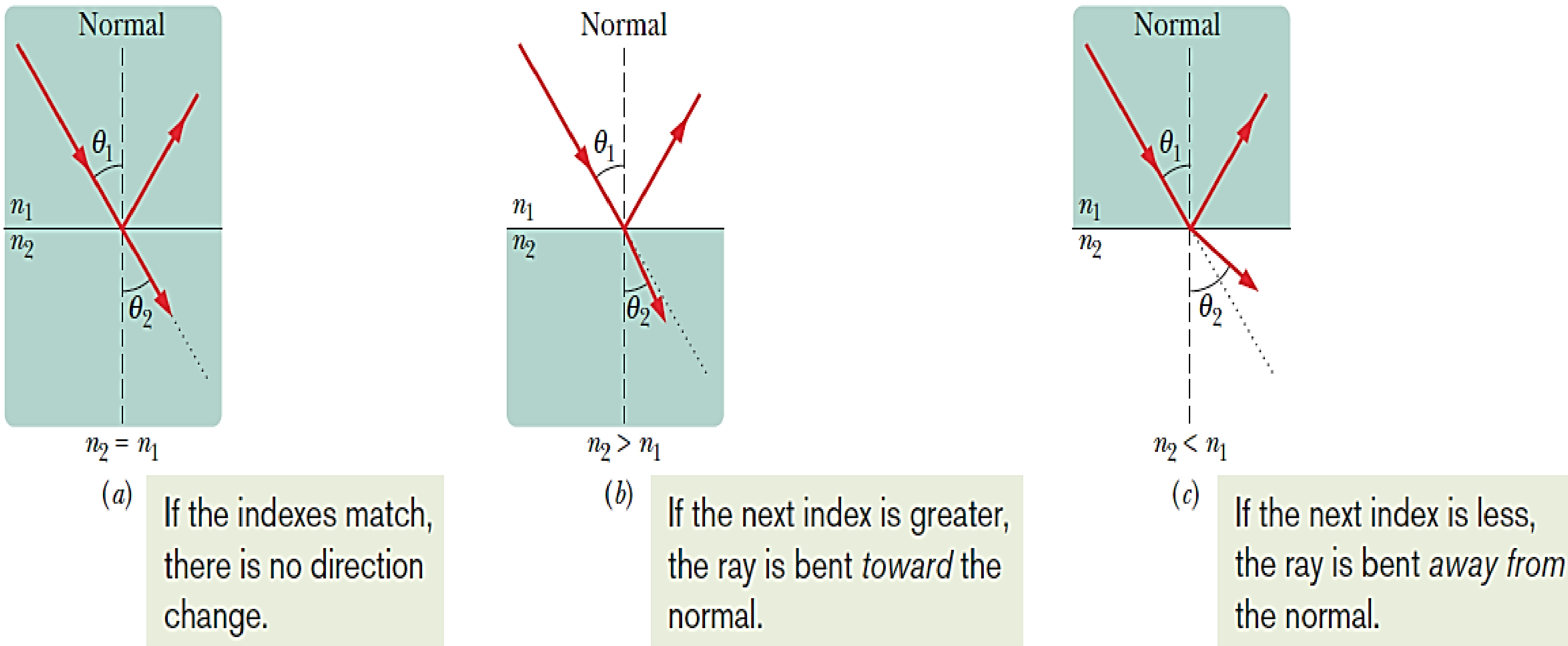


Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2. (*a*) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (*b*) toward the normal when $n_2 > n_1$ and (*c*) away from the normal when $n_2 < n_1$.

Chromatic Dispersion:

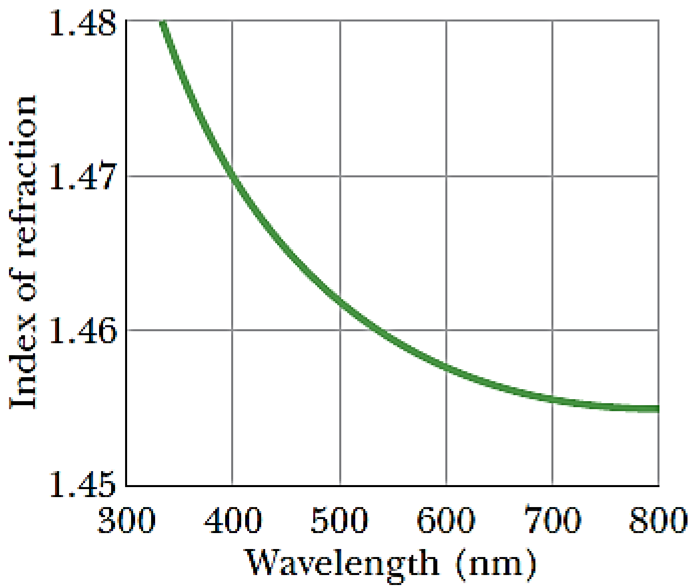


Fig. 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

The index of refraction *n* encountered by light in any medium except vacuum depends on the wavelength of the light.

The dependence of *n* on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction.

This spreading of light is called **chromatic dispersion**.

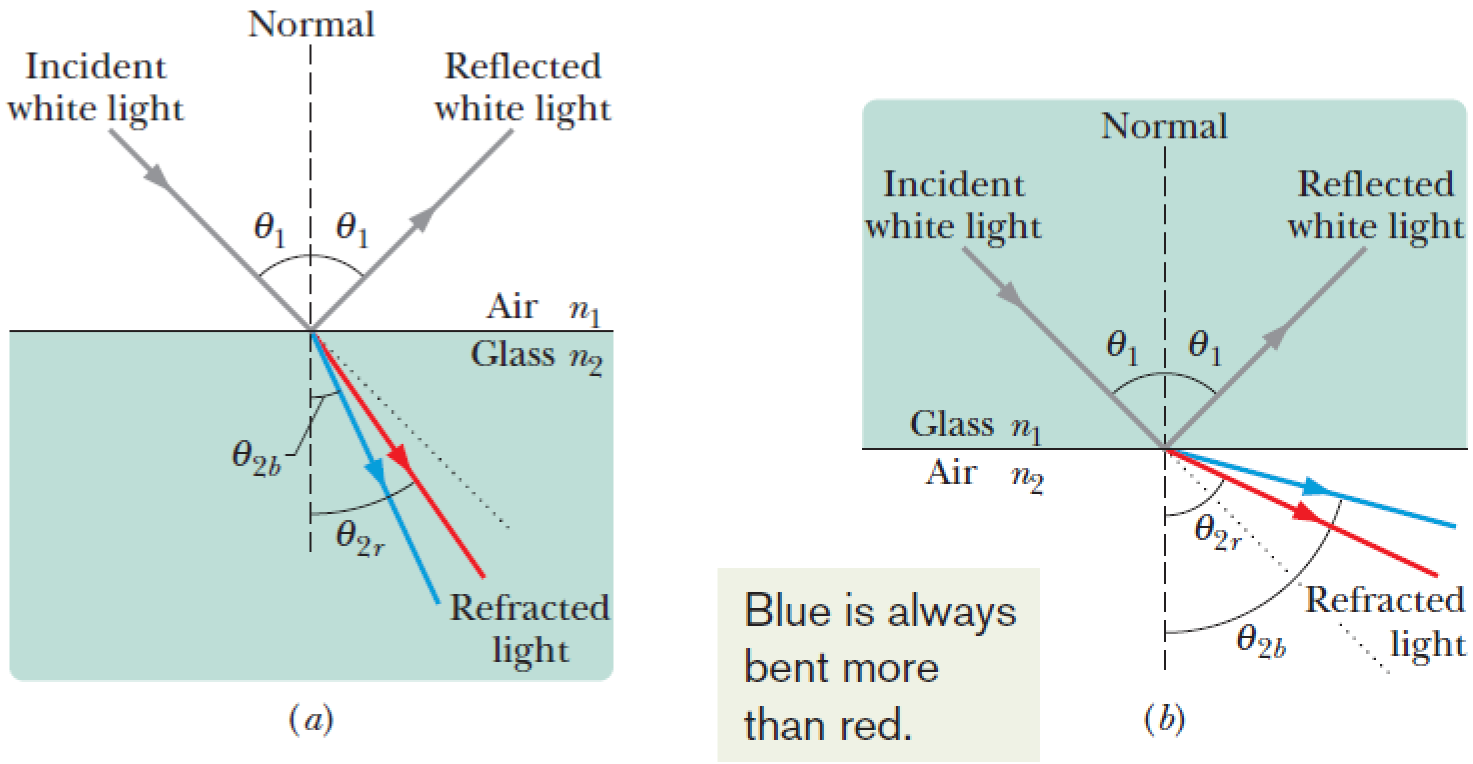


Fig. 33-19 Chromatic dispersion of white light. The blue component is bent more than the red component. (*a*) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (*b*) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.

Chromatic Dispersion and Rainbow:

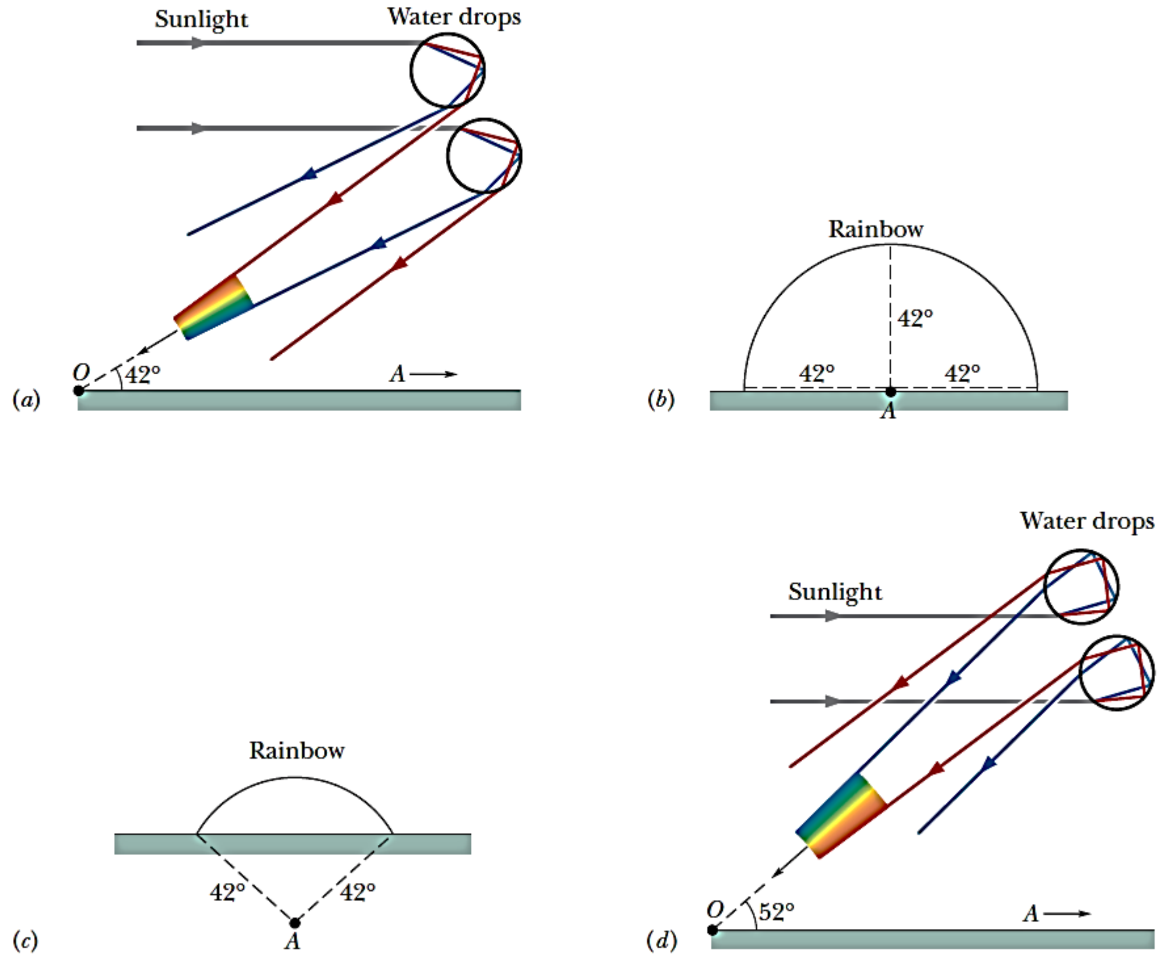
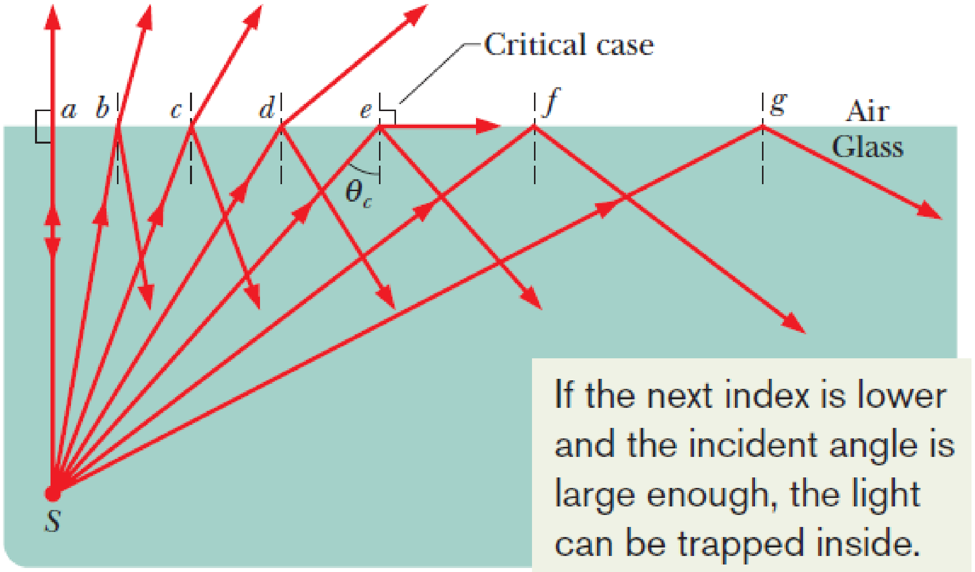


Fig. 33-21 (*a*) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point *A* is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of *A*. (*b*) Drops at 42° from *A* in any direction can contribute to the rainbow. (*c*) The rainbow arc when the Sun is higher (and thus *A* is lower). (*d*) The separation of colors leading to a secondary rainbow.

Total Internal Reflection:

Fig. 33-23 (*a*) Total internal reflection of light from a point source S in glass occurs for all angles of incidence greater than the critical angle θ_c. At the critical angle, the refracted ray points along the air–glass interface. (*b*) A source in a tank of water. *(Ken Kay/Fundamental Photographs)*



(*a*)

(*b*)

For angles of incidence larger than θ_c, such as for rays *f* and *g*, there is no refracted ray and all the light is reflected; this effect is called **total internal reflection.**

For the critical angle, $$n_1 \sin \theta_c = n_2 \sin 90°,$$

Which means that $$\theta_c = \sin^{-1} \frac{n_2}{n_1} \qquad \text{(critical angle).}$$

Polarization by Reflection:

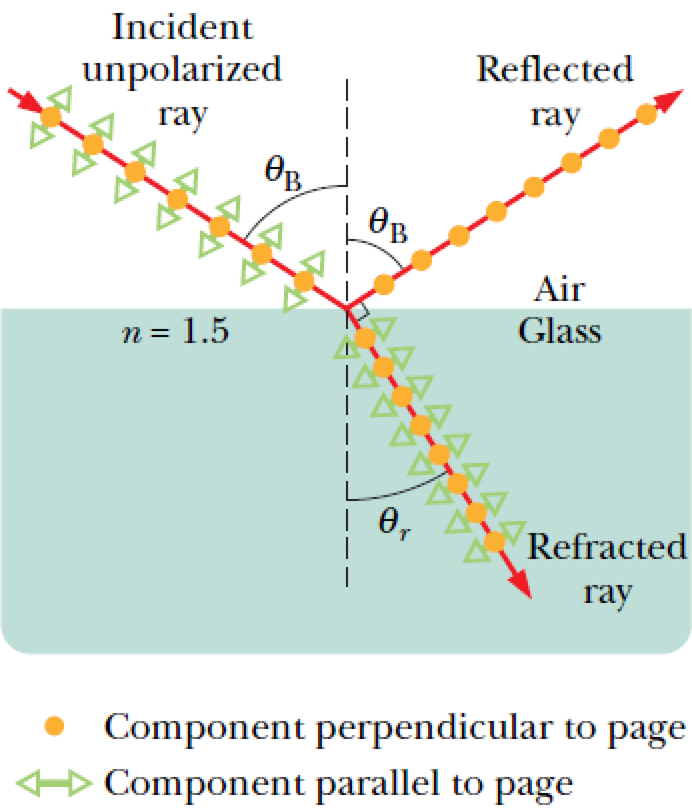


Fig. 33-25 A ray of unpolarized light in air is incident on a glass surface at the Brewster angle θ_B. The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

$$\theta_B + \theta_r = 90°.$$

$$n_1 \sin \theta_B = n_2 \sin \theta_r.$$

$$n_1 \sin \theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos \theta_B,$$

$$\boxed{\theta_B = \tan^{-1} \frac{n_2}{n_1} \qquad \text{(Brewster angle).}}$$